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THEORETICAL ARTICLE

A new reliable performance evaluation model: IFB-IER–DEA

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Abstract
Data envelopment analysis (DEA) is a well-known performance evaluation model for measuring the relative efficiency of multiple homogeneous decision making units (DMUs). Input/output of the DMUs can be uncertain especially when they are obtained from subjective human judgments. In this paper, Evidential Reasoning approach with interval assessment score and linguistic interval fuzzy belief degree (IFB-IER approach) is proposed to deal with the uncertainties. This approach does not restrict experts to give a precise point and provides more reliable and realistic performance evaluation. Eventually, the IFB-IER data are converted to standard interval data. Then, the efficiency of the DMUs is calculated using interval DEA model and Anderson Peterson model is applied to rank them. Finally, a numerical example is given to illustrate the validity and applicability of the proposed model.

Keywords Performance evaluation · DEA · Evidential Reasoning approach · IFB-IER approach · Anderson Peterson

1 Introduction

Data Envelopment Analysis (DEA) is classified as a non-parametric model to measure the performance of decision making units (DMUs). First, DEA model was proposed by Charnes et al. [1] and Banker et al. [2] extended it to consider variable return to scale. This model uses a fractional mathematical programming for evaluating the relative efficiency of several DMUs with multiple inputs and outputs. Numerous theoretical

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developments of DEA have been published since its introduction in 1978 (see Cook and Seiford [3]). Furthermore, DEA has been applied in various areas, such as education [4, 5], manufacturing [6–8], telecommunication [9, 10], transportation [11–13], healthcare [14, 15], financial [16, 17] and supply chain [18, 19]. Several reviewed papers were also published to explain the different models of the DEA and their applications [3, 20–22].

In the real world, some inputs and outputs of DEA model are uncertain especially when they are obtained from subjective human judgments. To handle these uncertainties, fuzzy logic has been applied in DEA. Several papers have been published in which fuzzy DEA has been developed and applied such as Kao and Liu [23–25], Saati et al. [26], and Wang et al. [27]. Kao and Liu [23–25] applied α level sets and Zadeh's extension principle to transform fuzzy data into interval and to build a family of Crisp DEA models. Saati et al. [26] adopted α level set approach to define the fuzzy DEA model as a possibilistic programming problem and to transform it into an interval programming. Their proposed model could be solved as a crisp linear programming. Wang et al. [27] proposed two new models to formulate fuzzy DEA model as a linear programming model and to determine fuzzy efficiencies of DMUs. Also, Jahanshahloo et al. [28] suggested a fuzzy ranking method in DEA.

To consider belief degree of experts about uncertain information and control more uncertainties such as incomplete/missing data ignored in the fuzzy uncertainty, Evidential Reasoning (ER) approach has been applied. This approach uses a set of assessment scores with different belief degrees to reflect opinions [29]. Wong et al. [30] used ER approach with qualitative assessment score in DEA model to measure performance of UK retail banks. Also, Yang et al. [31] provided a new cross-efficiency aggregation method in DEA models using the ER approach to reflect experts' value judgments. Despite the success obtained from employing ER approach in DEA model, there are some limitations such as ignoring uncertainty of belief degrees. Due to judging by interval data is more common and to overcome drawbacks of ER approach, ER approach with interval assessment score and linguistic interval fuzzy belief degree (IFB-IER approach) in DEA that called IFB-IER–DEA is proposed. The IFB-IER–DEA model handles the uncertainties in DEA more effective and allows people to express their subjective opinions freely resulting in more accurate evaluation. This model is solved by converting the IFB-IER data to the interval data.

The rest of the paper is organized as follows:

Section 2 studies concepts of DEA model, ER and IFB-IER approaches. Section 3 introduces the extended DEA model using the IFB-IER approach. Section 4 illustrates a numerical example of the proposed model to evaluate assumed project manager's performance. Finally, the paper is concluded in Sect. 5.

2 Preliminaries

2.1 Data envelopment analysis (DEA)

Suppose that there are n DMUs where jth DMU ($j = 1, \ldots, n$) produces s outputs ($y_{rj}(r = 1, \ldots, s)$) using m inputs, ($x_{ij}(i = 1, \ldots, m)$). The following model is used to evaluate oth DMU ($o = 1, \ldots, n$) [1]:

$$\begin{aligned} &\max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &\quad \text{s.t.} \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, j = 1, \dots, n \\ &\quad\quad u_r, v_i \ge \varepsilon,\ i = 1, \dots, m, r = 1, \dots, s \end{aligned} \tag{1}$$

where the objective function is the relative efficiency of oth DMU, and ε is a small positive value (non-Archimedean). In addition, V_i and U_r are decision variables and the weights of ith input and rth output, respectively. This model is known as constant return to scale (CCR) model and is the most popular DEA model. This programming can be easily converted to a simple linear programming by normalizing the numerator or denominator of the objective function [1]. The linear CCR model as the basic model of this paper is presented below.

$$\begin{aligned} &max \sum_{r=1}^{s} u_r y_{ro} \\ &\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, \dots, n \\ &\qquad \sum_{i=1}^{m} v_i x_{io} = 1 \\ &\qquad u_r, v_i \ge \varepsilon \quad i = 1, \dots, m, r = 1, \dots, s \end{aligned} \tag{2}$$

One of the main concerns of DEA models is managing uncertainties in the input and output variables. In this regard, many researchers extended DEA methods using fuzzy logic to answer such needs [23–27]. However, the main issue in the reviewed models is ignoring belief degree of fuzzy information obtained from human judgments. Other limitation of the fuzzy uncertainty is that it does not cover unawareness state. These limitations can be rectified by Evidential Reasoning approach.

2.2 Evidential reasoning (ER) approach

Multiple criteria decision problems in many cases require decision maker's subjective and intuitive judgments. The existence of different types of uncertainties, including incomplete and imprecise information is inevitable and should be considered [32]. Furthermore, decision makers don't always have enough knowledge and experience to give opinions or information may be inaccessible. ER approach that is based on multi attribute evaluation [33] and Dempster Shafer (D–S) theory of evidence can support all these problems and reduces the uncertainties [34]. Another advantage of the approach is aggregating different opinions as a set of belief structure using Dempster's combination rule [35]. First, this approach was used in artificial intelligence and expert systems as evidence modeling technique under uncertainty [29] and recently has been used in different fields of management and engineering such as FMEA [36, 37], QFD [38], supply chain [39], healthcare [40], etc.

In ER approach, data are expressed in belief structure including a set of assessment scores and different belief degrees. In the belief structure, assessment score may not be known precisely, but may be estimated to belong to intervals that is also common. So, assessing with interval score ER (IER) approach seems more functional.

Procedure of IER approach is summarized as follows:

Assume L experts are selected to evaluate performance based on several criteria. Each expert has a relative weight ($\lambda_\ell > \circ, \ell = 1, \dots, L$) so that $\sum_{\ell=1}^{L} \lambda_\ell = 1$ and they express their opinions using the belief structure of IER approach. Let $\{(H_{pq}, \beta_{pq}^{l}), p = 1, \dots, N, q = p, \dots, N\}$ be the belief structure provided by expert l where H_{pp} for p = 1 to N are the crisp scores defined for evaluation, H_{pq} for p = 1 to N and q = p + 1 to N are intervals between H_{pp} *and* H_{qq} *and* β_{pq}^{l} are belief degrees [38]. In order to explain clearly, assumed scoring scale is shown in Table 1. Also, possible scoring matrix is defined as follows:

$$\text{Possible scoring matrix:} \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \\ & H_{22} & H_{23} & H_{24} & H_{25} \\ & & H_{33} & H_{34} & H_{35} \\ & & & H_{44} & H_{45} \\ & & & & H_{55} \end{bmatrix} = \begin{bmatrix} 1 & 1-3 & 1-5 & 1-7 & 1-9 \\ & 3 & 3-5 & 3-7 & 3-9 \\ & & 5 & 5-7 & 5-9 \\ & & & 7 & 7-9 \\ & & & & 9 \end{bmatrix}$$

For example, an expert evaluates criteria to be good with the belief degree of 30% and average with the belief degree of 70%. This assessment can be modeled as {(7, 30%), (5, 70%)}. If an expert evaluates the criteria between excellent and good with the belief degree of 40% and average with the belief degree of 50%, this assessment can be characterized as {(7–9, 40%), (5, 50%)}. In the first assessment, the total belief degree is 100%, but in the second, the total degree is not 100% and called incomplete assessment. The remaining (10%) belief degree represents a lack of information and can be assigned to any of the scoring scale 1–9 [34]. The last state may happen, is unawareness about criteria. It can be characterized as {(1–9), 100%}.

The belief structures can be converted to interval data by calculating the expected values (E(S)). For example, the three belief structures can be modeled by expected values in the following way:

$$\{(7,\ 30\%),\ (5,\ 70\%)\} \rightarrow 7 \times 30\% + 5 \times 70\% = 5.6 = [5.6-5.6]$$

Table 1 Assumed scoring scale for evaluation

State	Score
Excellent	9
Good	7
Average	5
Poor	3
Worst	1

$$\{(7-9,\ 40\%),\ (5,\ 50\%)\} \rightarrow [7-9] \times 40\% \ + \ 5 \times 50\% \ + \ [1-9] \times 10\% \ = \ [5.4-7]$$

$$\{(1-9,\ 100\%)\} \rightarrow [1-9] \times 100\% \ = \ [1-9]$$

Integrating the experts' beliefs can be effective to decrease errors of the assessment. The collective assessment of criteria i can be expressed as the weighted sum of the expected values of the L experts as follows [38]:

$$E(S_i) = \sum_{l=1}^{L} \lambda_l E(S_i^l) \tag{3}$$

where λ_l is the relative weight of expert l and $E(S_i^l)$ is expected value of expert l's belief about criteria i.

Despite all the strengths of IER, this approach has one obvious drawback. In this approach, people are restricted to express their belief degrees to precise number. While, they may not always be confident. So, applying a new approach is required. For this purpose, IFB-IER approach is proposed.

2.3 IFB-IER approach

In this approach, we allow belief degrees to be vague instead of precise number and interval fuzzy number will be used to characterize vague belief degrees. Since, the experts cannot distinguish between adjacent belief degrees, using defined linguistic variables is more appropriate.

In this approach, data are expressed as an ordered pair of (A, B) denoted as IFB-IER data. A is an interval estimation of a value of X and B represents the belief degree in linguistic terms that both interval and fuzzy numbers are assigned to it. Table 2 and Fig. 1 show the possible linguistic variables for belief degrees and interval fuzzy number assigned to them.

Finally, the data under this uncertainty is converted to the standard interval data and applying them in the DEA models is usual. The calculations of the conversion are summarized as follows:

Suppose we have the data under IFB-IER uncertainty in the form of (A, B). A is interval assessment score and B is a linguistic belief degree that a triangular fuzzy number ((a, b, c)) is assigned to it. First, we calculate α-cut of the assigned fuzzy number (B_α) using Eq. (4) that results in the fuzzy number being converted to an interval number.

$$B = (a,\ b,\ c) \rightarrow B_\alpha = \ [(b-a) * \alpha + a,\ (b-c) * \alpha + c] \tag{4}$$

Table 3 indicates α-cut of the fuzzy numbers assigned to the linguistic variables per different values of α.

After calculating the α-cut values, the two interval data A and B are multiplied using Eq. (5) and final interval data per different values of α are obtained [41].

$$A = \ [a_1, a_2],\ B = [b_1, b_2] \text{ and } a_1, b_1 \geq 0 \rightarrow A.B = \ [a_1.b_1, a_2.b_2] \tag{5}$$

Table 2 The possible linguistic variables and interval fuzzy numbers assigned to them

Linguistic variable	Abbreviation	Assigned number	Linguistic variable	Assigned number	Linguistic variable	Assigned number	Linguistic variable	Assigned number
Very low	VL	(0.1, 0.2, 0.3)	VL–L	[0.2–0.3]	L–M	[0.3–0.5]	ML–VH	[0.4–0.8]
Low	L	(0.2, 0.3, 0.4)	VL–ML	[0.2–0.4]	L–MH	[0.3–0.6]	M–MH	[0.5–0.6]
Medium low	ML	(0.3, 0.4, 0.5)	VL–M	[0.2–0.5]	L–H	[0.3–0.7]	M–H	[0.5–0.7]
Medium	M	(0.4, 0.5, 0.6)	VL–MH	[0.2–0.6]	L–VH	[0.3–0.8]	M–VH	[0.5–0.8]
Medium high	MH	(0.5, 0.6, 0.7)	VL–H	[0.2–0.7]	ML–M	[0.4–0.5]	MH–H	[0.6–0.7]
High	H	(0.6, 0.7, 0.8)	VL–VH	[0.2–0.8]	ML–MH	[0.4–0.6]	MH–VH	[0.6–0.8]
Very high	VH	(0.7, 0.8, 0.9)	L–ML	[0.3–0.4]	ML–H	[0.4–0.7]	H–VH	[0.7–0.8]

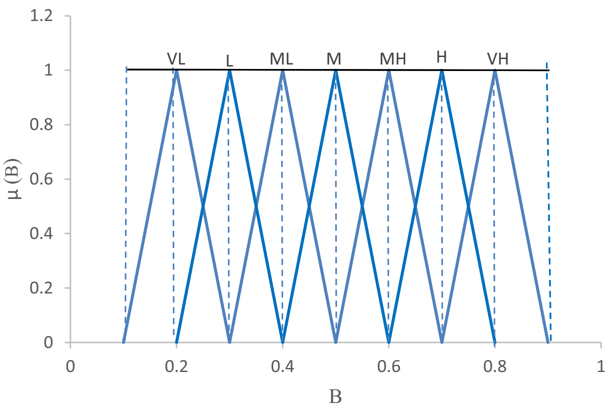

Fig. 1 Interval fuzzy numbers assigned to B

Also, when linguistic belief degree (B) is interval, the conversion will be obtained through the use of Eq. (5), too.

3 IFB-IER–DEA model

Assume that X_{ijl} and Y_{rjl} are input and output of jth DMU according to the lth expert's belief as IFB-IER numbers and $X_{ijl} = \left(A^X_{ijl}, B^X_{ijl}\right)$ $(i=1, \ldots, m, j=1, \ldots, n, l=1, \ldots, L)$ and $Y_{rjl} = \left(A^Y_{rjl}, B^Y_{rjl}\right)$ $(r=1, \ldots, s, j=1, \ldots, n, l=1, \ldots, L)$. In order to apply the information in the form of IFB-IER in the DEA, the following framework is proposed in Fig. 2:

In the final stage, according to Lotfi et al. [42], the relative efficiency of the DMU_o with interval input $([X^L_{ij}, X^U_{ij}])$ and interval output $([Y^L_{rj}, Y^U_{rj}])$ is obtained by solving the following models:

Table 3 α-cut of the linguistic variables per different values of α

Variable	α-cut	$\alpha=0$	$\alpha=0.25$	$\alpha=0.5$	$\alpha=0.75$	$\alpha=1$
VL	$[0.1\ \alpha+0.1, -0.1\ \alpha+0.3]$	[0.1, 0.3]	[0.12, 0.27]	[0.15, 0.25]	[0.17, 0.22]	[0.2, 0.2]
L	$[0.1\ \alpha+0.2, -0.1\ \alpha+0.4]$	[0.2, 0.4]	[0.22, 0.37]	[0.25, 0.35]	[0.27, 0.32]	[0.3, 0.3]
ML	$[0.1\ \alpha+0.3, -0.1\ \alpha+0.5]$	[0.3, 0.5]	[0.32, 0.47]	[0.35, 0.45]	[0.37, 0.42]	[0.4, 0.4]
M	$[0.1\ \alpha+0.4, -0.1\ \alpha+0.6]$	[0.4, 0.6]	[0.42, 0.57]	[0.45, 0.55]	[0.47, 0.52]	[0.5, 0.5]
MH	$[0.1\ \alpha+0.5, -0.1\ \alpha+0.7]$	[0.5, 0.7]	[0.52, 0.67]	[0.55, 0.65]	[0.57, 0.62]	0.6
H	$[0.1\ \alpha+0.6, -0.1\ \alpha+0.8]$	[0.6, 0.8]	[0.62, 0.77]	[0.65, 0.75]	[0.67, 0.72]	0.7
VH	$[0.1\ \alpha+0.7, -0.1\ \alpha+0.9]$	[0.7, 0.9]	[0.72, 0.87]	[0.75, 0.85]	[0.77, 0.82]	0.8

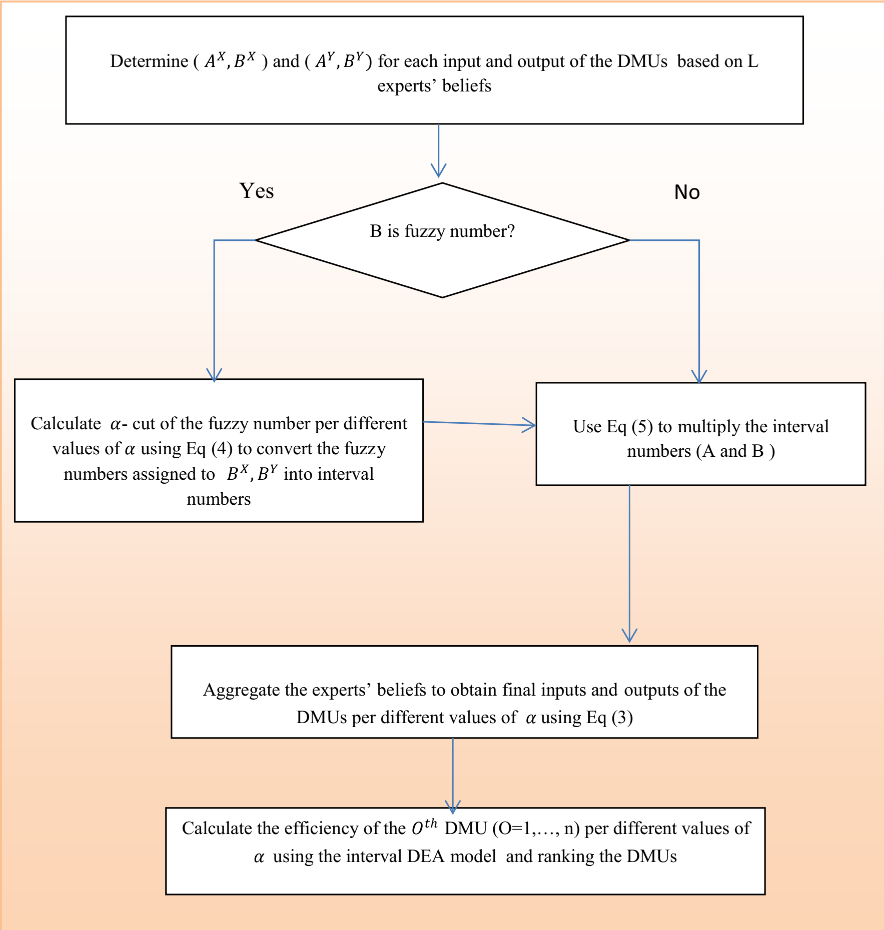

Fig. 2 Framework of the proposed IFB-IER–DEA model

$$
\begin{aligned}
& e_O^U = e_O^1 = max \sum_{r=1}^{s} u_r y_{ro}^U \\
& \qquad \text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \le 0 \quad j = 1, \dots, n, j \ne o \\
& \qquad \sum_{r=1}^{s} u_r y_{ro}^U - \sum_{i=1}^{m} v_i x_{io}^L \le 0 \\
& \qquad \sum_{i=1}^{m} v_i x_{io}^L = 1 \\
& \qquad u_r, v_i \ge \varepsilon \quad i = 1, \dots, m, r = 1, \dots, s
\end{aligned}
\tag{I}
$$

$$
\begin{aligned}
e_O^L = e_O^2 = & \max \sum_{r=1}^{s} u_r y_{ro}^L \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0 \quad j = 1, \ldots, n, j \ne o \\
& \sum_{r=1}^{s} u_r y_{ro}^L - \sum_{i=1}^{m} v_i x_{io}^U \le 0 \\
& \sum_{i=1}^{m} v_i x_{io}^U = 1 \\
& u_r, v_i \ge \varepsilon \quad i = 1, \ldots, m, r = 1, \ldots, s
\end{aligned}
\tag{II}
$$

In the model (I), DMU_o is in its most favorable condition (upper bound of outputs and lower bound of inputs) and the other DMUs are in their least favorable conditions. While, in the model (II), DMU_o is in its least favorable condition and the others are in their most favorable conditions. So, the DMUs are categorized into three groups. Group 1 includes DMUs which are efficient under any condition ($e_j^1 = 1, e_j^2 = 1$). The DMUs of the group 2 are efficient only in their most favorable conditions (($e_j^1 = 1, e_j^2 < 1$) and group 3 consists of the DMUs which are inefficient both in their most favorable and least favorable conditions ($e_j^1 < 1, e_j^2 < 1$).

Therefore, the group 1 is the most efficient group and all the DMUs of the group 3 are inefficient. About the group 2, we cannot determine its efficiency or inefficiency. But, generally we can say that the group 2 has a higher rank compared to the group 3. For ranking the DMUs in each group, computing efficiency values of them under the other two conditions as the following models (III) and (IV) can be helpful because more information provides more accurate results [42]:

(a) The DMU_o is in its most favorable condition, and the other DMUs are in their most favorable conditions.

$$
\begin{aligned}
e_O^3 = & \max \sum_{r=1}^{s} u_r y_{ro}^U \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0 \quad j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_i x_{io}^L = 1
\end{aligned}
$$

$$
u_r, v_i \ge \varepsilon \quad i = 1, \ldots, m, r = 1, \ldots, s
\tag{III}
$$

(b) The DMU_O is in its least favorable condition, and the others are in their least favorable conditions, too.

$$
\begin{aligned}
e_O^4 = max \sum_{r=1}^{s} u_r y_{ro}^L \\
\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \le 0 \quad j = 1, \dots, n \\
\sum_{i=1}^{m} v_i x_{io}^U = 1 \\
u_r, v_i \ge \varepsilon \quad i = 1, \dots, m, r = 1, \dots, s
\end{aligned} \tag{IV}
$$

By calculating the weighted average of the efficiency values under the four conditions ($e_O^1, \dots, e_O^4$), many of the DMUs can be ranked, but still some DMUs continue to be efficient. In order to cover these units in ranking, Anderson Peterson (AP) model [43] is solved for each DMU under the four conditions as follows.

$$
\begin{aligned}
\theta_O^1 = max \sum_{r=1}^{s} u_r y_{ro}^U \\
\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \le 0 \quad j = 1, \dots, n, j \ne o \\
\sum_{i=1}^{m} v_i x_{io}^L = 1 \\
u_r, v_i \ge \varepsilon \quad i = 1, \dots, m, r = 1, \dots, s
\end{aligned} \tag{V}
$$

$$
\begin{aligned}
\theta_O^2 = max \sum_{r=1}^{s} u_r y_{ro}^L \\
\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0 \quad j = 1, \dots, n, j \ne o \\
\sum_{i=1}^{m} v_i x_{io}^U = 1 \\
u_r, v_i \ge \varepsilon \quad i = 1, \dots, m, r = 1, \dots, s
\end{aligned} \tag{VI}
$$

Table 4 PMBOK criteria

Output	Description	Output	Description
1	Integration management	5	Quality management
2	Scope management	6	Haman resource management
3	Time management	7	Communication management
4	Cost management	8	Logistic management

$$
\begin{aligned}
\theta_O^3 = max & \sum_{r=1}^{s} u_r y_{ro}^U \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 \quad j = 1, \ldots, n, j \neq o \\
& \sum_{i=1}^{m} v_i x_{io}^L = 1 \\
& u_r, v_i \geq \varepsilon \quad i = 1, \ldots, m, r = 1, \ldots, s
\end{aligned}
\tag{VII}
$$

$$
\begin{aligned}
\theta_O^4 = max & \sum_{r=1}^{s} u_r y_{ro}^L \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \leq 0 \quad j = 1, \ldots, n, j \neq o \\
& \sum_{i=1}^{m} v_i x_{io}^U = 1 \\
& u_r, v_i \geq \varepsilon \quad i = 1, \ldots, m, r = 1, \ldots, s
\end{aligned}
\tag{VIII}
$$

Table 5 IFB-IER data based on the experts′ beliefs about criteria 1

DMU	*Expert*1(20%)	*Expert*2(20%)	*Expert*3(20%)	*Expert*4(20%)	*Expert*5(20%)
1	{(5, M), (1–3, M)}	{(3–5, MH)}	{(5, VH)}	Unknown	{(3, VH)}
2	{(7–9, M)}	{(9, H), (7, L)}	{(3–7, H)}	{(5–7, MH)}	{(5, MH)}
3	{(1–5, M)}	{(3, ML), (5, M)}	{3–9, VH)}	{(5–9, ML)}	{(3–5, H)}
4	{(7, M), (9, ML)}	{(7, VH)}	{(7–9, M)}	{(7–9, ML)}	{(7, VL)}
5	{(5, H)}	{3, VL), (5–7, L)}	{(7, M)}	{(1–7, MH)}	{(5, L)}
6	{(1, M)}	Unknown	{(3, ML), (5, MH)}	{(1–3, L)}	{(3–7, ML)}
7	{(7–9, ML)}	{(7, H)}	{(7–9, MH)}	{(5–7, L)}	{(9, M)}
8	{(3, MH), (5, L)}	{(1–5, M)}	{(3, M)}	{(5–9, H)}	{(3–9, H)}
9	{(5–7, ML)}	{(3–7, ML), (9, MH)}	{(7–9, MH)}	{(7, M), (9, ML)}	{(9, VH)}
10	{(3–7, VH)}	{(7, H)}	Unknown	{(7–9, VH)}	{(7, MH)}
11	{(1, MH)}	{(1–3, VH)}	{(1–5, ML)}	{(1–5, L)}	{(3, M)}
12	{(5–7, H)}	{(3, H)}	{(3, VH)}	{(5, VL), (3, VH)}	{(5, MH)}
13	{(1, MH)}	{(1–3, M)}	{(1, H), (3, L)}	{(3, ML)}	{(3, MH)}
14	Unknown	{(1–3, M)}	{(3–5, H)}	{(3, VH)}	{(3–5, M)}
15	{(7–9, M)}	{(7–9, ML)}	{(9, VH)}	{(9, M)}	{(7, ML), (9, MH)}
16	{(7, MH)}	{(9, M)}	{(7, M), (9, ML)}	{(7–9, M)}	{(9, M)}
17	{(5–7, M)}	{(5, M), (7, M)}	{(5, MH)}	{(7, VH)}	{(7, M)}
18	{(1–3, M)}	{(1, VH)}	{(1–5, M)}	{(1–3, ML)}	{(3, M)}
19	{(3–5, H)}	{(3–5, VL)}	{(5, H)}	{(3, MH)}	{(1–5, M)}
20	{(7, M)}	{(7–9, M)}	{(3–7, VH)}	{(3–5, M)}	{(7, MH)}

As can be seen, in these models, the constraints corresponding to the DMU_o are removed. By calculating the weighted average of the obtained efficiency values for each DMU $\left(\bar{\theta}_j\right)$, they can be ranked fully.

In summary, the strengths of the proposed IFB-IER–DEA model are as follows:

- Supporting more uncertainties (such as incomplete/missing data) in DEA model than conventional approaches to control uncertainty like fuzzy.
- Considering belief degree of experts about uncertain data in DEA model.
- Overcoming drawbacks of IER approach in restricting people to express their belief degrees as precise number.
- Assessing with interval score and linguistic belief degree that is easy and functional for experts.
- Reflecting real experts' opinions resulting in accurate performance evaluation.

4 Numerical example

In this section, we apply the proposed IFB-IER–DEA model to evaluate the performance of 20 assumed project managers based on PMBOK (project management body of knowledge) standard that is the most famous global standard in project

Table 6 Expected values of each belief and integrated for output 1 per $\alpha = 1$

DMU	*Expert*1(20%)	*Expert*2(20%)	*Expert*3(20%)	*Expert*4(20%)	*Expert*5(20%)	Integrated belief
1	[3–4]	[2.2–6.6]	[4.2–5.8]	[1–9]	[2.6–4.2]	[2.6–5.92]
2	[4–9]	8.4 = [8.4–8.4]	[2.4–7.6]	[3.4–7.8]	[3.4–6.6]	[4.32–7.88]
3	[1–7]	[3.8–4.6]	[2.6–9]	[2.6–9]	[2.4–6.2]	[2.48–7.16]
4	[7.2–8]	[5.8–7.4]	[4–9]	[3.4–9]	[2.2–8.6]	[4.52–8.4]
5	[3.8–6.2]	[2.1–2.7]	[4–8]	[1–7.8]	[2.2–7.8]	[2.62–6.5]
6	[1–5]	[1–9]	4.2 = [4.2–4.2]	[1–7.2]	[1.8–8.2]	[1.8–6.72]
7	[3.4–9]	[5.2–7.6]	[4.9–9]	[2.2–8.4]	[5–9]	[4.14–8.6]
8	[3.4–4.2]	[1–7]	[2–6]	[3.8–9]	[2.4–9]	[2.52–7.04]
9	[2.6–8.2]	[6.6–8.2]	[4.6–9]	[7.2–8]	[7.4–9]	[5.68–8.48]
10	[2.6–7.4]	[5.2–7.6]	[1–9]	[5.8–9]	[4.6–7.8]	[3.84–8.16]
11	[1–4.2]	[1–4.2]	[1–7.4]	[1–7.8]	[2–6]	[1.2–5.92]
12	[3.8–7.6]	[2.4–4.8]	[2.4–4.2]	3.4 = [3.4–3.4]	[3.4–6.6]	[3.08–5.32]
13	[1–4.2]	[1–6]	1.6 = [1.6–1.6]	[1.8–6.6]	[2.2–5.4]	[1.52–4.76]
14	[1–9]	[1–6]	[2.4–6.2]	[2.6–4.2]	[2–7]	[1.8–6.48]
15	[4–9]	[3.4–9]	[7.4–9]	[5–9]	8.2 = [8.2–8.2]	[5.6–8.84]
16	[4.6–7.8]	[5–9]	[7.2–8]	[4–9]	[5–9]	[5.16–8.56]
17	[3–8]	6 = [6–6]	[3.4–6.6]	[5.8–7.4]	[4–8]	[4.44–7.2]
18	[1–6]	[1–2.6]	[1–7]	[1–6.6]	[2–6]	[1.2–5.64]
19	[2.4–6.2]	[1.4–8.2]	[3.8–6.2]	[2.2–5.4]	[1–7]	[2.16–6.6]
20	[4–8]	[4–9]	[2.6–7.4]	[2–7]	[4.6–7.8]	[3.44–7.84]

Table 7 Final values of output 1 for each DMU per $\alpha = 0$, 0.25, 0.5, 0.75

DMU	$\alpha=0$	$\alpha=0.25$	$\alpha=0.5$	$\alpha=0.75$
1	[2.1, 4.9]	[2.16, 4.77]	[2.27, 4.65]	[2.33, 4.56]
2	[3.28, 5.8]	[3.52, 1.82]	[3.66, 5.44]	[3.78, 5.17]
3	[1.74, 4.8]	[1.82, 4.59]	[1.94, 4.46]	[2.02, 4.24]
4	[3.2, 5.4]	[3.37, 5.11]	[3.64, 4.92]	[3.81, 4.63]
5	[1.72, 3.7]	[1.82, 3.55]	[1.98, 3.42]	[2.08, 3.21]
6	[1.18, 3.86]	[1.23, 3.74]	[1.31, 3.67]	[1.36, 3.55]
7	[2.88, 4.9]	[3.02, 4.67]	[3.23, 4.51]	[3.37, 4.26]
8	[1.78, 4.66]	[1.86, 4.45]	[1.98, 4.32]	[2.06, 4.11]
9	[4.44, 7.28]	[4.62, 6.93]	[5.04, 6.71]	[5.12, 6.36]
10	[3.14, 6.78]	[3.23, 7.21]	[3.38, 6.48]	[3.47, 6.3]
11	[0.58, 1.94]	[0.608, 1.83]	[0.65, 1.77]	[0.67, 1.66]
12	[2.4, 3.68]	[2.49, 3.52]	[2.74, 3.53]	[2.73, 3.25]
13	[0.9, 5.4]	[0.91, 1.48]	[1.01, 1.36]	[1.06, 1.38]
14	[1.3, 4.1]	[1.34, 3.89]	[1.4, 3.94]	[1.44, 3.84]
15	[4.28, 6.64]	[4.47, 6.32]	[4.76, 6.12]	[4.95, 5.8]
16	[3.8, 5.96]	[3.99, 5.66]	[4.28, 5.46]	[4.47, 5.16]
17	[3.4, 5.08]	[3.54, 4.85]	[3.76, 4.7]	[3.89, 4.47]
18	[0.6, 1.8]	[0.62, 1.71]	[0.67, 1.65]	[0.64, 1.56]
19	[1.4, 2.92]	[1.46, 2.84]	[1.55, 2.69]	[1.61, 2.55]
20	[2.48, 4.76]	[2.58, 4.55]	[2.75, 4.41]	[2.85, 4.2]

management [44]. First we collect 5 experts' beliefs, so that, they are in IFB-IER structure and the experts have equal relative weights (0.2). Each manager is treated as an individual DMU and PMBOK criteria, listed in Table 4, are considered as outputs of the model. Because of output nature of all PMBOK criteria, input of the model is assumed to be 1.

First, the experts express their assessment scores based on the scoring scale in the Table 1 and defined possible matrix and to express their belief degrees; they use the Table 2. For instance, Table 5 shows the experts' beliefs about criteria 1.

In the second step, the fuzzy numbers, assigned to some belief degrees, are converted to interval numbers by calculating α-cut of them per different values of α according to Table 3. Then, the expected values of each belief per different values of α are calculated through Eq. (5). For belief degrees that interval data are assigned to them, these values are obtained directly using the Eq. (5) and Table 2. Then, the beliefs are integrated according to the results obtained and the weights of the experts. The results for the first output per $\alpha = 1$ are shown in Table 6.

The integrated belief per other values of α is calculated in this way. Table 7 shows final value of output 1 per $\alpha = 0, 0.25, 0.5, 0.75$.

Value of other outputs for each DMU is obtained similarly. Final outputs per $\alpha = 1$ are shown in Table 8.

In the next step, the efficiency of the DMUs is calculated under the mentioned conditions using the models I–IV and DMUs are grouped. Finally, they are ranked as explained in the previous section. The results per $\alpha = 1$ can be seen in the Table 9.

Table 8 Interval outputs of the DMUs per $\alpha = 1$

DMU	Y_{1j}	Y_{2j}	Y_{3j}	Y_{4j}	Y_{5j}	Y_{6j}	Y_{7j}	Y_{8j}
1	[2.6–5.92]	[4–7.24]	[3.2–6.92]	[4.2–8.28]	[2.24–6.04]	[1–7.84]	[2.4–6.2]	[3.72–8.08]
2	[4.32–7.88]	[4.72–8.68]	[2.16–7.2]	[2.16–7.28]	[6–8.36]	[3.64–7.8]	[2.2–7]	[3.12–8.4]
3	[2.48–7.16]	[1.28–5.44]	[2.52–5.44]	[4.64–8.16]	[4.24–7.8]	[1.76–5.8]	[2.68–7.32]	[2.88–7.64]
4	[4.52–8.4]	[2.88–7.2]	[3.76–8.28]	[3.76–7.8]	[2.48–8.64]	[4.02–7.7]	[4.84–7.24]	[3.82–8.18]
5	[2.62–6.5]	[2.2–6.16]	[3.36–8.36]	[4.1–7.24]	[3.64–7.4]	[4–7.56]	[5.84–7.52]	[2.2–6.92]
6	[1.8–6.72]	[2.72–7.32]	[3.68–7.52]	[1.48–4.72]	[3.72–7.88]	[3.28–7.6]	[3.64–6.32]	[5.16–8.32]
7	[4.14–8.6]	[4.52–8.12]	[2.72–7.8]	[3–7.16]	[3.76–7.8]	[2.16–4.76]	[4.52–7.88]	[2.04–7]
8	[2.52–7.04]	[4.76–7.96]	[1.64–6.24]	[2.68–7]	[5.44–8.08]	[4.36–7.2]	[3.08–6.52]	[1.8–7.16]
9	[5.68–8.48]	[2.08–7.44]	[1.76–6.32]	[2.64–7.52]	[2.72–6.64]	[3.76–6]	[4.88–7.76]	[4.04–7.2]
10	[3.84–8.16]	[5.04–8.64]	[3.08–8.04]	[2.28–6.76]	[4.04–7.24]	[3.52–7.16]	[3.04–7.8]	[3.68–8.72]
11	[1.2–5.92]	[3.12–6.68]	[2.32–6.84]	[4.96–8.4]	[2.96–7.44]	[6.2–7.96]	[2.64–6.48]	[3.72–6.36]
12	[3.08–5.32]	[1.24–5.6]	[3.24–6.88]	[2.96–5.6]	[2–5.72]	[2.88–5.48]	[4.68–7.92]	[3.44–8.76]
13	[1.52–4.76]	[2.64–7.88]	[2.72–6.44]	[2.24–5.6]	[1.52–6.88]	[4.24–6.48]	[3.12–8.44]	[2.12–6.32]
14	[1.8–6.48]	[3.16–7.4]	[3.6–5.52]	[4.96–8.88]	[2.76–8.8]	[2.64–6.8]	[2.44–6.8]	[2.6–8.52]
15	[5.6–8.84]	[3.52–9]	[5.44–7.76]	[2.56–7.24]	[1–4.96]	[3.88–6.36]	[3.64–7.16]	[3.48–6.76]
16	[5.16–8.56]	[2.48–6.6]	[3.52–7.56]	[2–6.28]	[3.2–7.2]	[3.32–6.4]	[2.44–7.88]	[3.64–8.52]
17	[4.44–7.2]	[2.76–6.32]	[4.36–6.96]	[3.36–7.8]	[2.56–6.36]	[5.04–7.76]	[2.68–7.52]	[3.16–7.76]
18	[1.2–5.64]	[5.24–8.52]	[4.03–8.68]	[2.56–6.32]	[5.12–8.36]	[3.16–6.96]	[4.2–6.88]	[2.04–5.84]
19	[2.16–6.6]	[4.24–8.04]	[3.84–8.2]	[1.28–6]	[3.2–8.76]	[4.04–8.4]	[3.4–8.2]	[3.96–8.76]
20	[3.44–7.84]	[3.8–7.16]	[2.32–4.68]	[3.4–6.88]	[3.52–7.64]	[2.04–4.76]	[2.4–7.72]	[2.56–7.36]

Table 9 Efficiency values, classification and ranking of the DMUs per $\alpha = 1$

DMU$_j$	e_j^1	e_j^2	e_j^3	e_j^4	Group	θ_j^1	θ_j^2	θ_j^3	θ_j^4	$\bar{\theta}_j$	Rank
1	1	0.5052	1	1	2	2.149	0.5052	1.0271	1.064	1.186	16
2	1	0.6818	1	1	2	2.306	0.6818	1.053	1.277	1.329	1
3	1	0.5225	1	1	2	2.169	0.5225	1.0018	1.117	1.202	14
4	1	0.601	1	1	2	2.418	0.601	1.0843	1.12	1.305	5
5	1	0.6995	1	1	2	2.167	0.6995	1.007	1.23	1.275	7
6	1	0.589	0.951	1	2	2.201	0.589	0.951	1.29	1.257	9
7	1	0.5628	1	1	2	2.203	0.5628	1.0327	1.111	1.227	12
8	1	0.6194	0.9512	1	2	2.082	0.6194	0.9512	1.07	1.180	17
9	1	0.651	1	1	2	2.111	0.651	1.0134	1.168	1.235	11
10	1	0.5725	1	1	2	2.314	0.5725	1.0504	1.108	1.261	8
11	1	0.7513	1	1	2	2.169	0.7513	1.0317	1.349	1.325	2
12	1	0.5608	1	0.942	2	1.9748	0.5608	1	0.942	1.119	18
13	1	0.5047	1	0.841	2	1.9583	0.5047	1.0292	0.8419	1.083	20
14	1	0.5904	1	1	2	2.3455	0.5904	1.1044	1.145	1.296	6
15	1	0.6638	1	1	2	2.2685	0.6638	1.0636	1.263	1.314	3
16	1	0.5864	1	1	2	2.1555	0.5864	1.0298	1.021	1.197	15
17	1	0.617	1	1	2	2.0835	0.617	1.0205	1.1	1.205	13
18	1	0.6089	1	1	2	2.3373	0.6089	1.0587	1.247	1.312	4
19	1	0.5184	1	1	2	2.2947	0.5184	1.1059	1.038	1.238	10
20	1	0.4479	0.9786	0.965	2	2.0568	0.4479	0.9786	0.965	1.112	19

Regarding these results, we can see that the DMUs are efficient in their most favorable conditions and inefficient in their least favorable conditions. So, all of them are placed in the group 2 and those with higher $\bar{\theta}$, have a higher rank.

For other values of α, efficiency values and ranking of the DMUs are calculated similarly. Table 10 presents final efficiency values and ranking of the DMUs per $\alpha = 0$, 0.25, 0. 5 and 0.75.

5 Conclusion

Data envelopment analysis (DEA) is classified as a non-parametric performance evaluation, which many studies have been done to develop it. Input and output variables in DEA have some degree of uncertainty that should be considered. This study proposes the IFB-IER approach to eliminate the weakness of the ER approach and deal with interval uncertainty and linguistic belief degree including both fuzzy and interval data. Also, this approach in the DEA models provides a more realistic and accurate evaluation. Finally, the proposed model is converted to the traditional interval DEA and to obtain the efficiency values and rank the DMUs, we used the proposed method by Lotfi et al. [42] and the AP model. The numerical example is given to illustrate the steps of the model. For further researches, the algorithm can be applied to the other DEA models such as Banker, Charnes and Copper model

Table 10 Average efficiency values and ranking of the DMUs per $\alpha = 0, 0.25, 0.5, 0.75$

$\mathrm{DMU_j}$	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$	
	$\bar{\theta}_j$	Rank	$\bar{\theta}_j$	Rank	$\bar{\theta}_j$	Rank	$\bar{\theta}_j$	Rank
1	1.203	12	1.225	12	1.218	14	1.195	15
2	1.338	3	1.335	3	1.332	4	1.332	1
3	1.257	7	1.248	9	1.235	3	1.216	13
4	1.352	2	1.366	1	1.348	1	1.329	2
5	1.308	4	1.282	4	1.279	6	1.274	8
6	1.273	6	1.26	7	1.262	9	1.259	9
7	1.247	10	1.263	6	1.272	7	1.279	7
8	1.252	8	1.224	13	1.208	15	1.177	17
9	1.094	18	1.125	19	1.173	17	1.192	16
10	1.09	19	0.128	18	0.17	18	0.227	12
11	1.365	1	1.34	2	1.336	2	1.319	3
12	1.154	16	1.182	16	1.194	16	1.171	18
13	0.91	20	0.94	20	0.099	20	1.056	20
14	1.173	14	1.208	14	1.254	10	1.285	6
15	1.196	13	1.239	11	1.28	5	1.312	4
16	1.25	9	1.241	10	1.232	13	1.203	14
17	1.231	11	1.258	8	1.27	8	1.248	10
18	1.163	15	1.19	15	1.249	11	1.287	5
19	1.293	5	1.265	5	1.241	12	1.235	11
20	1.152	17	1.169	17	1.154	19	1.138	19

(BCC), slacks Based Measure model (SBM) or Network DEA (NDEA) model. Another potential extension of this research is employing other ranking methods such as cross efficiency instead of AP.

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